Meta-Stable Brane Configurations by Adding an Orientifold-Plane to Giveon-Kutasov

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Abstract

In hep-th/0703135, they have found the type IIA intersecting brane configuration where there exist three NS5-branes, D4-branes and anti-D4-branes. By analyzing the gravitational interaction for the D4-branes in the background of the NS5-branes, the phase structures in different regions of the parameter space were studied in the context of classical string theory. In this paper, by adding the orientifold 4-plane and 6-plane to the above brane configuration, we describe the intersecting brane configurations of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of these gauge theories.

1 Introduction

It is known that the NS5-brane configuration in type IIA string theory where there exist two types of NS5-branes, i.e., NS5-brane(012345) and NS5'-brane(012389), preserves $\mathcal{N} = 2$ supersymmetry in four dimensions [1]. By adding D4-branes(01236), that are suspended between the NS5-brane and the NS5'-brane, and anti-D4-branes($\overline{D4}$-branes), that are suspended between the NS5-brane and the other NS5'-brane, into this system, the supersymmetry is broken [2]. As the distance between the two NS5'-branes along one of the longitudinal directions of the NS5-brane becomes zero, this brane configuration with D4- and $\overline{D4}$-branes can decay and the geometric misalignment between flavor D4-branes, which can be interpreted as a nontrivial F-term condition in the gauge theory side, arises. Due to the presence of NS5-brane in this system, there is an attractive force between the tilted D4-branes and NS5-brane. The explicit computation of DBI action for these D4-branes in the background of NS5-brane is done by the work of [2] recently and this effect of the gravitational attraction leads to a curve for tilted D4-branes rather than a straight line. The meta-stable vacua of [3] appear in some region of parameter space.

In this paper, we focus on the new meta-stable brane configurations by adding an orientifold 4-plane and an orientifold 6-plane to the above brane configuration studied by [2], along the line of [4, 5, 6, 7]. When the former is added, no extra NS5-branes or D-branes are needed. However, when the latter is added, the extra NS5-branes or D-branes into the above brane configuration are needed in order to have a product gauge group. All of these examples have very simple dual magnetic superpotentials which make it easier to analyze meta-stable brane configurations.

In section 2, we review the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $Sp(N_c) \times SO(2N_c')$ gauge theory with a bifundamental and deform this theory by adding the mass term for the bifundamental. Then we construct the dual magnetic theory which is $\mathcal{N} = 1$ $Sp(\widetilde{N}_c) \times SO(2N_c')$ gauge theory with corresponding dual matter as well as gauge singlet for the first gauge group factor. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. We also discuss the dual magnetic theory which is $\mathcal{N} = 1$ $Sp(N_c) \times SO(2\widetilde{N}_c')$ gauge theory briefly.

In section 3, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N_c')$ gauge theory with matters and deform this theory by adding the mass term for the bifundamentals. Then we construct the dual magnetic theory which is $\mathcal{N} = 1$ $SU(\widetilde{N}_c) \times SU(N_c')$ gauge theory with corresponding dual matters as

well as gauge singlet for the first gauge group factor. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. We also consider the same gauge theory with different matters and describe the nonsupersymmetric meta-stable brane configuration from the dual magnetic theory which is $\mathcal{N}=1$ $SU(\widetilde{N}_c) \times SU(N_c')$ gauge theory.

In section 4, we make some comments for the future directions.

2 When an O4-plane is added

In this section, we add an orientifold 4-plane to the type IIA brane configurations of [2] and construct new meta-stable brane configurations.

2.1 Electric theory

The type IIA brane configuration corresponding to $\mathcal{N}=1$ supersymmetric gauge theory with gauge group

$$Sp(N_c) \times SO(2N_c') \tag{2.1}$$

and a bifundamental X that is in the representation $(\mathbf{2N_c}, \mathbf{2N_c'})$ under the gauge group (2.1) can be described by a middle NS5-brane(012345), the left $NS5_L'$-brane(012389) and the right $NS5_R'$-brane(012389), $2N_c$- and $2N_c'$-color D4-branes(01236) as well as an $O4^+$-plane(01236) and an $O4^-$-plane(01236) we should add. We take the arbitrary number of color D4-branes with the constraint $N_c' \geq N_c + 2$. The $O4^{\pm}$-planes act as $(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9)$ as usual and they have RR charge ± 1 playing the role of ± 1 D4-brane. The bifundamental X corresponds to 4-4 strings connecting the $2N_c$-color D4-branes with $2N_c'$-color D4-branes.

The middle NS5-brane is located at $x^6 = 0$ and we denote the x^6 coordinates for the $NS5_L'$-brane and $NS5_R'$-brane by $x^6 = -y_1 (< 0)$ and $x^6 = y_2 (> 0)$ respectively, along the line of [2]. The $2N_c$ D4-branes and $O4^+$-plane are suspended between the middle NS5-brane and $NS5_R'$-brane while the $2N_c'$ D4-branes and $O4^-$-plane are suspended between the $NS5_L'$-brane and the middle NS5-brane. Moreover, there exist $O4^+$-plane(which will extend to $x^6 = -\infty$) to the left side of $NS5_L'$-brane and $O4^-$-plane(which will extend to $x^6 = +\infty$) to the right side of $NS5_R'$-brane. We draw this brane configuration in Figure 1A for the vanishing mass for the bifundamental X by inserting the appropriate $O4^{\pm}$-planes into the brane configuration of [2] [1]. See also the relevant works appeared in [8, 9, 10, 11]. The gauge group and matter

[1]This is equivalent to the reduced brane configuration of [11] if we remove D6-branes from [11] completely.

content of [2] are changed as above by orientifolding procedure to that theory.

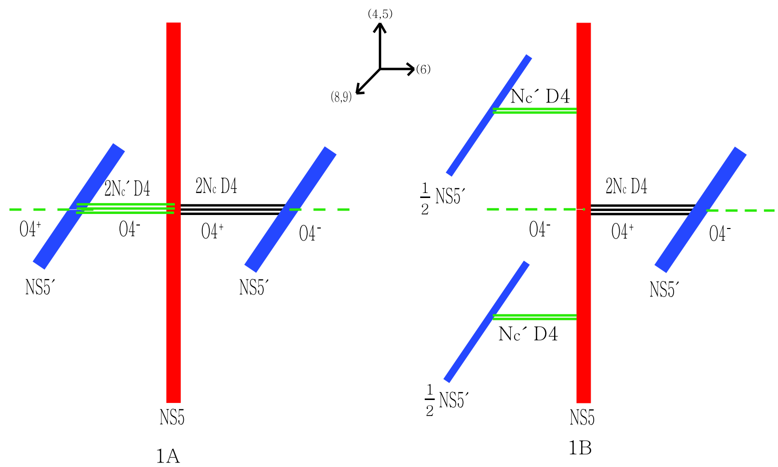

Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_c) \times SO(2N_c')$ and a bifundamental X with vanishing(1A) and nonvanishing(1B) mass for the bifundamental. The bifundamental X corresponds to 4-4 strings connecting the $2N_c$-color D4-branes with $2N_c'$-color D4-branes.

The gauge couplings of $Sp(N_c)$ and $SO(2N_c')$ are given by a string coupling constant g_s, a string scale ℓ_s and the x^6 coordinates y_i for two NS5'-branes through

$$g_{Sp}^2 = \frac{g_s \ell_s}{y_2}, \qquad g_{SO}^2 = \frac{g_s \ell_s}{y_1}$$

respectively. As y_1 goes to ∞ implying the change of the relative strength for the two gauge couplings, the $SO(2N_c')$ gauge group becomes a global symmetry and the theory leads to SQCD with the gauge group $Sp(N_c)$ and N_c' flavors(or $2N_c'$ fields) in the fundamental representation. On the other hand, the opposite limit $y_2 \to \infty$ leads to SQCD with the gauge group $SO(2N_c')$ with $2N_c$ fields in the fundamental representation.

There is no superpotential in Figure 1A. Let us deform this gauge theory. Displacing the two NS5'-branes relative each other in the $v \equiv x^4 + ix^5$ direction corresponds to turning on a quadratic mass-deformed superpotential for the bifundamental X as follows:

$$W = mXX \, (\equiv m\Phi) \tag{2.2}$$

where a symplectic metric that has antisymmetric color indices [11] is assumed in the $Sp(N_c)$ gauge group indices for XX, the Φ is a meson field and the mass m is given by geometrically

$$m = \frac{\Delta x}{2\pi\alpha'} \left(= \frac{\Delta x}{\ell_s^2} \right).$$

Half of $NS5'_L$-brane together with N'_c color D4-branes is moving to the $+v$ direction and half of $NS5'_L$-brane together with other N'_c color D4-branes is moving to $-v$ direction due to the O4-plane for fixed $NS5'_R$-brane during this mass deformation. See also [12] for the splitting of branes on orientifold planes in the general context. The splitting of $NS5'_R$-brane for fixed $NS5'_L$-brane can be applied also and will be explained later in subsection 2.4. Then the x^5 coordinate($\equiv x$) of $NS5'_R$-brane is equal to zero and the x^5 coordinates of each half $NS5'_L$-brane are given by $\pm\Delta x$ respectively. Giving an expectation value to the meson field Φ corresponds to recombination of $2N_c$- and $2N'_c$- color D4-branes, which will become $2N_c$-color D4-branes because $N'_c > N_c$, in Figure 1A such that they are suspended between the $NS5'_L$-brane and the $NS5'_R$-brane and pushing them into the $w \equiv x^8 + ix^9$ direction. Now we draw this brane configuration in Figure 1B for nonvanishing mass for the bifundamental X by moving half of $NS5'_L$-brane with N'_c color D4-branes to the $+v$ direction and their mirrors to $-v$ direction.

2.2 Magnetic theory

By applying the Seiberg dual to the $Sp(N_c)$ factor in (2.1), the two $NS5'_{L,R}$-branes can be located at the same side of the NS5-brane. Starting from Figure 1B and moving the NS5-brane to the right all the way past the $NS5'_R$-brane, one obtains the Figure 2A. Before arriving at the Figure 2A, there exists an intermediate step where the N'_c D4-branes are connecting between half $NS5'_L$-brane and $NS5'_R$-brane(and their mirrors) and $2\widetilde{N}_c$ D4-branes connecting between $NS5'_R$-brane and NS5-brane. By introducing $2N'_c$ D4-branes and $2N'_c$ anti-D4-branes between $NS5'_R$-brane and NS5-brane, reconnecting half of the former with the N'_c D4-branes that are connecting between half $NS5'_L$-brane and $NS5'_R$-brane and moving those combined D4-branes to v-direction(and their mirrors to $-v$ direction), one gets the final Figure 2A where we are left with $2(N'_c - \widetilde{N}_c)$ anti-D4-branes between $NS5'_R$-brane and NS5-brane.

Then the gauge group is given by

$$Sp(\widetilde{N}_c = N'_c - N_c - 2) \times SO(2N'_c) \tag{2.3}$$

where the number of dual color is obtained from the linking number counting, as done in [11]. The matter contents are the field Y in the bifundamental representation $(\mathbf{2\widetilde{N}_c}, \mathbf{2N'_c})$ under the dual gauge group (2.3) and the gauge-singlet Φ for the first dual gauge group in the adjoint representation for the second dual gauge group, i.e., $(\mathbf{1}, \mathbf{N'_c}(\mathbf{2N'_c} - \mathbf{1}))$ under the dual gauge group (2.3). These matter fields introduce a cubic superpotential which is an interaction between dual "quarks" Y and a meson Φ.

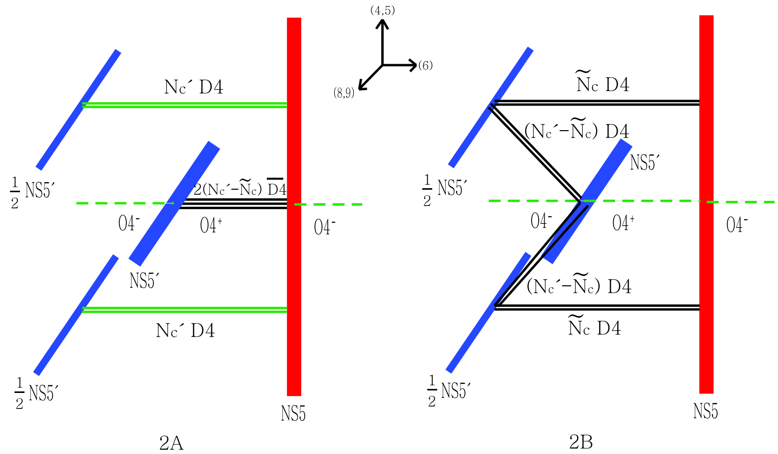

Figure 2: The magnetic brane configuration corresponding to Figure 1B with D4- and $\overline{D4}$-branes(2A) and with a misalignment between D4-branes(2B) when the NS5'-branes are close to each other.

Then the dual magnetic superpotential, by adding the mass term (2.2) for the bifundamental X, which can be interpreted as a linear term in the meson Φ, to this cubic superpotential, is given by

$$W_{dual} = \Phi YY + m\Phi. \tag{2.4}$$

This can be seen from the equation (2.2) of [11] by removing the terms of D6-branes in electric and magnetic theories. Of course, the brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 2A by recombination between half NS5'-branes together with color D4-branes via pushing them into the origin $v = 0$. Then the number of dual colors for D4-branes becomes $2N_c'$ between two NS5'-branes and $2\widetilde{N}_c$ between $NS5'_R$-brane and NS5-brane. Or starting from Figure 1A and moving the NS5-brane to the right all the way past the $NS5'_R$-brane, one also obtains the corresponding magnetic brane configuration for massless bifundamental.

The brane configuration in Figure 2A is stable as long as the distance Δx between the upper half $NS5'_L$-brane and $NS5'_R$-brane is large, as in [2]. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 2B with bending effect of tilted D4-branes connecting half NS5'-brane and NS5'-brane. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.3) and superpotential (2.4). The difference between the energies of the configurations of Figure 2A and Figure 2B by evaluating the lengths of D4-branes in order to determine the true ground state can be obtained. When the two half NS5'-branes are replaced by two coincident D6-branes, the brane configuration of Figure 2B is the same as

the one studied in [4, 13].

According to the result of [2], the flavor D4-branes of straight brane configuration of Figure 3 of [2] can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. For example, see the Figure 6 of [2] for explicit curve, obtained by extremizing the DBI action, connecting two NS5'-branes. The correct phase transition of the classical string theory in the background and which one is the correct ground state of the system among the Figure 2 and the Figure 6 of [2] depends on the parameters y_i, the locations of two NS5'-branes, and Δx, the relative displacement between two NS5'-branes in the v-direction. For example, when the equal y_i is less than a string scale ℓ_s, for small Δx, the ground state is given by the brane configuration in Figure 6 of [2] while for larger Δx, the ground state is given by the brane configuration in Figure 2 of [2].

One can perform similar analysis in our brane configuration with an addition of O4-plane since one can take into account the behavior of parameters geometrically in the presence of O4-plane. Then the upper $(N_c'-\widetilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 2B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action, by following the procedure of [2]. Of course, their mirrors, the lower $(N_c'-\widetilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 2B can bend and their trajectory connecting two NS5'-branes should be preserved under the O4-plane, i.e., $\mathbf{Z}_2$ symmetric way, like as the symmetry property between the straight flavor D4-branes, when there is no gravitational interaction, under the O4-plane in Figure 2B. The correct choice for the ground state of the system depends on the parameters y_i and Δx. When the equal y_i is less than a string scale ℓ_s, for small Δx, the ground state is the brane configuration in Figure 2B with an appropriate bending effect and for larger Δx, the ground state is the brane configuration in Figure 2A.

In the Figure 2B, the background geometry by NS5-brane gives an attractive force between the upper tilted $(N_c'-\widetilde{N}_c=N_c+2)$ D4-branes and the NS5-brane. At first, we focus on the upper tilted D4-branes and later we'll describe its mirror, the lower tilted D4-branes in Figure 2B. In order to compute the upper bending curve for the D4-branes, i.e., a generalization of coincident straight lines due to an attractive force, connecting between the upper NS5'-brane and the middle NS5'-brane in Figure 2B explicitly, one has to analyze the DBI action for the upper tilted D4-branes in this background geometry. For example, see the ref. [14] for detailed explanations on brane dynamics near NS5-branes.

By following the procedure [2, 14], one can write down the DBI action for the upper tilted

D4-branes as, after inserting both the dilaton and the induced metric,

$$S(upper) = -(N_c + 2)\tau_4 \int dx \sqrt{\frac{1}{H(y)} + (\partial_x y)^2}, \qquad H(y) \equiv 1 + \frac{\ell_s^2}{y^2} \tag{2.5}$$

where τ_4 is the tension of the D4-brane, the induced NS B field vanishes [14] and the harmonic function $H(y)$ is the field strength of NS B field. In the present case, the l in $H(y)$ is equal to ℓ_s because we consider a single NS5-brane background. The coordinate r of [2] corresponds to the radial coordinate away from the NS5-brane in the transverse (6789) directions, in general, but this will become $r = y = x^6$ at $x^7 = x^8 = x^9 = 0$. The constant term $+2$ in the number of D4-branes, $(N_c + 2)$, appearing in front of (2.5) is due to the presence of O4-plane. This feature is a new fact, when we compare with the unitary case [2]. For the different O4-plane charge in the subsection 2.4, we'll see the opposite coefficient for the constant term -2.

Since the integrand of this action (2.5) does not depend on x explicitly, there exists a conserved constant quantity [2], through the Euler-Lagrange equation,

$$H(y)\sqrt{\frac{1}{H(y)} + (\partial_x y)^2} = C = const. \tag{2.6}$$

We are looking for a solution, by extremizing the DBI action (2.5), where the upper tilted D4-branes are described by a smooth coincident curve $y = y(x)$ connecting between the upper NS5'-brane and the middle NS5'-brane in Figure 2B. Let us consider the solution of this equation of motion (2.6) as a function of a relative distance between two NS5'-branes

$$\Delta x = x_2 - x_1 = x_2$$

where we put the x coordinate of the upper NS5'-brane in Figure 2B as $x = x_2$ while the x coordinate of the middle NS5'-brane as $x = x_1 = 0$. In other words, the O4-plane is located at $x = 0$. Then the mirror of the upper NS5'-brane, the lower NS5'-brane, is located at $x = -x_2$. It is evident that for nonzero Δx which corresponds to massive case, the solution for bending curve provides a deformation of the upper tilted $(N_c + 2)$ coincident straight D4-branes. At the minimum value of y, which is denoted by y_m, the above equation (2.6) leads to

$$C^2 = H(y_m) \tag{2.7}$$

since $\partial_x y$ at $y = y_m$ vanishes.

By using the separation of variables for (x, y) in (2.6) together with (2.7), one can write the integrals for the two intervals $0 \leq x \leq x_m$ and $x_m \leq x \leq x_2$ where the x_m is the corresponding x coordinate on the curve to $y = y_m$. The former corresponds to the interval

$y_1 \leq y \leq y_m$ while the latter does the interval $y_m \leq y \leq y_2$. We expect the same result as the one in [2] because the brane configuration consisting of two NS5'-branes, the NS5-brane and tilted D4-branes in Figure 2B is exactly the same as the Figure 6 of [2] with an interchange of horizontal and perpendicular coordinates and the exact results for the integrals turn out as follows [2]:

$$\begin{aligned}
\int_{y_m}^{y_1} dy \frac{H(y)}{\sqrt{H(y_m)-H(y)}} &= \frac{y_m}{\ell_s}\sqrt{y_1^2-y_m^2}+\ell_s\theta_1 = -\int_{x_m}^{0} dx = x_m, \\
\int_{y_m}^{y_2} dy \frac{H(y)}{\sqrt{H(y_m)-H(y)}} &= \frac{y_m}{\ell_s}\sqrt{y_2^2-y_m^2}+\ell_s\theta_2 = \int_{x_m}^{x_2} dx = x_2 - x_m
\end{aligned} \tag{2.8}$$

where we introduce the angles

$$\cos\theta_i \equiv \frac{y_m}{y_i}, \qquad 0 \leq \theta_i \leq \frac{\pi}{2}, \tag{2.9}$$

for $\theta_i \to 0$, the y_m goes to y_i, for $\theta_i \to \frac{\pi}{2}$, the y_m approaches to zero, and unfortunately, we use the same variables y_i as the one in an electric theory and in Figure 1. In other words, the y coodinate for the upper NS5'-brane in Figure 2B is given by $y = y_2$ while the y coodinate for the middle NS5'-brane in Figure 2B is given by $y = y_1$, as in [2]. The positive direction of y in Figure 2B is directed to the left hand side of NS5-brane whose y coordinate is zero. Note that the numerator $H(y)$ in the above integrands consists of two parts, i.e., constant term and y-dependent term through (2.5). The constant term of numerator $H(y)$ in the integrals gives rise to the $1/\ell_s$ term in the middle of (2.8) while the y-dependent term of numerator $H(y)$ contributes to the ℓ_s term in the middle of (2.8), after the y-integrations.

Now we have the explicit relation between Δx and y_i and y_m, by little algebra for the trigonometric functions, and by adding the two integral results above (2.8), as in [2], the relative distance between two NS5'-branes for the curve, depending on y_i and y_m, is given by

$$\Delta x(upper) = x_2 = \frac{1}{2\ell_s}\left(y_1^2 \sin 2\theta_1 + y_2^2 \sin 2\theta_2\right) + \ell_s\left(\theta_1+\theta_2\right). \tag{2.10}$$

This is invariant under $y_i \to -y_i$ and $y_m \to -y_m$. Note that θ_i is also invariant under these transformations, by (2.9). When $\theta_i = 0$, then $\Delta x = 0$ which is for the massless case and for $\theta_i = \frac{\pi}{2}$, the Δx is equal to $\pi \ell_s$ that corresponds to the configuration of Figure 2A.

Moreover, the energy of the Figure 2B with bending effect for the upper D4-branes is given by

$$E_{curved}(upper) = -S(upper) \tag{2.11}$$

together with (2.5). By using the relation (2.7) and change the integration over x into the y variable with (2.6), this leads to $E_{curved}(upper) = (N_c+2)\tau_4\sqrt{H(y_m)}\int dy \frac{1}{\sqrt{H(y_m)-H(y)}}$. Then

one arrives at the following expression for the energy for D4-branes, as in [2], by adding the results of (2.8) that contain $1/\ell_s$ term, as we mentioned before,

$$E_{curved}(upper) = (N_c+2)\tau_4 \frac{\sqrt{H(y_m)}}{2\ell_s}\left(y_1^2 \sin 2\theta_1 + y_2^2 \sin 2\theta_2\right). \tag{2.12}$$

In other words, the first two terms in (2.10) appear in (2.12) as a factor. This energy (2.12) is also invariant under $y_i \to -y_i$ and $y_m \to -y_m$ with (2.9). Note that there exists a constant term $+2$ coming from the O4-plane, in the overall coefficient of (2.12).

So far, we have only considered the contributions, i.e., the explicit curve connecting two NS5'-branes and the energy of the configuration in that background, from the bending effect of upper tilted D4-branes in Figure 2B. Now we can compute the contributions from their mirrors, i.e., the lower tilted (N_c+2) D4-branes in Figure 2B. The DBI action for these D4-branes, $S(lower)$, is the same as above $S(upper)$ given in (2.5) since the number of D4-branes are the same and the background geometry is characterized by the same NS5-brane, implying that the induced metric and a dilaton are the same as before. That is,

$$S(lower) = S(upper) \tag{2.13}$$

with (2.5). It is straightfoward to see that there exist a conserved quantity (2.6) and a relation (2.7). Now the coordinate x of lower NS5'-brane in Figure 2B is given by $x = -x_2$, as mentioned before. Remember that the O4-plane action restricts the position of lower NS5'-brane in this particular way. Of course, the coordinate x of middle NS5'-brane is equal to $x = 0$. Also the y coordinate of lower NS5'-brane in Figure 2B is the same as $y = y_2$ for the upper NS5'-brane.

By using the separation of variables from (2.6) with (2.7), one can write the integrals for the two intervals $-x_m \leq x \leq 0$ and $-x_2 \leq x \leq -x_m$, as we did before. Note that the x values for the bending curve are all negative except the one of the middle NS5'-brane. It turns out the results are given by (2.8) and (2.9) again. Moreover the relative distance $\Delta x(lower) = 0 - (-x_2) = x_2$ between the middle NS5'-brane and the lower NS5'-brane is given by (2.10):

$$\Delta x(lower) = \Delta x(upper).$$

The $\mathbf{Z}_2$ symmetry by an O4-plane acting as $(x, y) \to (-x, y)$ reflects here if we take the absolute value for Δx. The two bending curves for the upper and lower tilted D4-branes are symmetric each other under O4-plane. Since the energy of the Figure 2B with bending effect for the lower tilted D4-branes is given by minus $S(lower)$ which is related to (2.13) and

further (2.11), eventually one obtains that

$$E_{curved}(lower) = E_{curved}(upper)$$

with (2.12).

Then the analysis of [2] can be done, using the results of both (2.10) which is exactly the same as the one in [2] and (2.12) which has different overall coefficient due to the O4-plane when we compare with the unitary case [2], for equal y_i's and unequal y_i's. Since the correct choice for the ground state of the system depends on the parameters y_i and Δx, once we understand the right phase structure for the upper tilted case given by (2.10) and (2.12), then the corresponding phase structure for its mirror, the lower tilted case, is satisfied automatically.

2.3 Gauge theory analysis at small Δx, a mass for the fundamental

The quantum corrections can be understood for small Δx by using the low energy field theory on the branes. The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N}=1$ supersymmetric gauge theory with gauge group (2.3) and the gauge couplings for the two gauge group factors are given by

$$g^2_{Sp,mag} = \frac{g_s \ell_s}{y_2}, \qquad g^2_{SO,mag} = \frac{g_s \ell_s}{(y_1 - y_2)}. \tag{2.14}$$

In the classical string theory, the gauge theory is weakly coupled, i.e., small g_s with fixed $\frac{y_i}{\ell_s}$. By tuning y_1 and y_2, one of the gauge couplings can be larger than the other.

The dual gauge theory has an adjoint Φ of $SO(2N_c')$, i.e., an antisymmetric matrix and bifundamental Y in the representation $(\mathbf{2\widetilde{N}_c}, \mathbf{2N_c'})$ under the dual gauge group (2.3) and the superpotential corresponding to Figures 2A and 2B is given by

$$W_{dual} = h\Phi YY - h\mu^2\Phi, \qquad h^2 = g^2_{SO,mag}$$

in the parametrization of [3] and we used the equation of (2.9) of [6] for the value of h. Here the mass parameter is given by

$$\mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}. \tag{2.15}$$

That is, the second term in the superpotential measures the separation of the NS5'-branes in the x direction. Then YY is a $2\widetilde{N}_c \times 2\widetilde{N}_c$ matrix where the second gauge group indices for two Y's are contracted with those of Φ while μ^2 is a $2N_c' \times 2N_c'$ antisymmetric matrix. Although the field Y itself is a fundamental in the second gauge group which is a different

feature, compared with the singlet representation for the usual quark coming from D6-branes [11], the product YY has the same representation with the product of quarks, $\widetilde{Q}\widetilde{Q}$ in the notation of [11]. Moreover, the second gauge group indices for the field Φ play the role of the flavor indices for the gauge singlet $S \equiv QQ$ in [11].

Therefore, the F-term equation, the derivative W_{dual} with respect to the meson field Φ cannot be satisfied if the $2N_c'$ exceeds $2\widetilde{N}_c$. So the supersymmetry is broken. That is, there are two equations from F-term conditions: $YY - \mu^2 = 0$ and $\Phi Y = 0$. Then the solutions for these are given by

$$< Y >= \begin{pmatrix} \mu \mathbf{1}_{2\widetilde{N}_c} \\ 0 \end{pmatrix}, \qquad < \Phi >= \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{(N_c' - \widetilde{N}_c)} \otimes i\sigma_2 \end{pmatrix}. \tag{2.16}$$

Then one can expand these fields around on a point (2.16), as in [3, 11] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{eff}^{(1)}$ for Φ_0 leads to the positive value for $m^2_{\Phi_0}$ implying that these vacua are stable.

By extremizing the low energy superpotential [3], the supersymmetric vacua occur at

$$< h\Phi >= \Lambda_1 \left(\frac{\mu}{\Lambda_1}\right)^{\frac{2(\widetilde{N}_c+1)}{(N_c'-\widetilde{N}_c-1)}} \mathbf{1}_{N_c'} \otimes i\sigma_2. \tag{2.17}$$

We are interested in the case where $N_c' > 3(\widetilde{N}_c + 1)$ so that $Sp(\widetilde{N}_c)$ gauge coupling is IR free. It becomes strongly coupled at the scale

$$\Lambda_1 = E_c \exp\left[\frac{8\pi^2 y_2}{(N_c' - 3(\widetilde{N}_c + 1))g_s \ell_s}\right] \tag{2.18}$$

where the expression (2.14) is used. Then, the condition that $< h\Phi >$ is much smaller than E_c implies, by plugging (2.15) and (2.18) into (2.17), that the gauge theory analysis is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C [2].

2.4 Other magnetic theory with same electric theory

By applying the Seiberg dual to the $SO(2N_c')$ factor in (2.1), the two $NS5'_{L,R}$-branes can be located at the right side of the NS5-brane. Starting from modified Figure 1B, where the x^5 coordinate of $NS5'_L$-brane is equal to zero and the x^5 coordinates of half $NS5'_R$-brane are $\pm\Delta x$, and moving the NS5-brane to the left all the way past the $NS5'_L$-brane, one obtains the magnetic brane configuration similar to Figure 2A. The gauge group is given by

$$Sp(N_c) \times SO(2\widetilde{N}_c' = 2N_c - 2N_c' + 4). \tag{2.19}$$

The matter contents are the field Y in the bifundamental representation $(\mathbf{2N_c}, \mathbf{2\widetilde{N}'_c})$ under the dual gauge group (2.19) and the gauge-singlet Φ for the second dual gauge group in the adjoint representation for the first dual gauge group, i.e., a symmetric matrix, $(\mathbf{N_c(2N_c+1)}, \mathbf{1})$ under the dual gauge group. The superpotential is the same as the one in (2.4) and the corresponding Figure 2B, which is exactly a reflection of Figure 2B with respect to the NS5-brane, i.e., all the D4-branes and NS5'-branes are located at the right hand side of NS5-brane, can be constructed similarly. The DBI analysis done in previous case can be obtained also in this case. The number of relevant D4-branes here is given by $(N'_c - 2)$ which plays the role of (N_c+2) in previous section.

The gauge couplings for the two gauge group factors are given by

$$g^2_{Sp,mag} = \frac{g_s \ell_s}{(y_2 - y_1)}, \qquad g^2_{SO,mag} = \frac{g_s \ell_s}{y_1}$$

and the superpotential corresponding to modified Figures 2A and 2B is given by

$$W_{dual} = h\Phi YY - h\mu^2\Phi, \qquad h^2 = g^2_{Sp,mag}$$

where the mass parameter μ^2 is given by (2.15). Then the solutions for these are given by

$$< Y >= \left(\begin{array}{c} \mu \mathbf{1}_{2\widetilde{N}'_c} \\ 0 \end{array} \right), \qquad < \Phi >= \left(\begin{array}{cc} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{2(N_c - \widetilde{N}'_c)} \end{array} \right).$$

At one loop, the effective potential $V^{(1)}_{eff}$ for Φ_0 leads to the positive value for $m^2_{\Phi_0}$ implying that these vacua are stable.

By extremizing the low energy superpotential [3], the supersymmetric vacua occur at

$$< h\Phi >= \Lambda_2 \left(\frac{\mu}{\Lambda_2} \right)^{\frac{2(\widetilde{N}'_c - 2)}{(N_c - \widetilde{N}'_c + 2)}} \mathbf{1}_{N_c}.$$

We are interested in the case where $N_c > 3(\widetilde{N}'_c - 2)$ so that $SO(2\widetilde{N}'_c)$ gauge coupling is IR free. It becomes strongly coupled at the scale

$$\Lambda_2 = E_c \exp \left[\frac{8\pi^2 y_1}{(N_c - 3(\widetilde{N}'_c - 2)) g_s \ell_s} \right].$$

Then, the condition that $< h\Phi >$ is much smaller than E_c implies that the gauge theory analysis is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C.

3 When an O6-plane is added

In this section, we add an orientifold 6-plane to the type IIA brane configurations of [2] together with two extra outer NS5-branes and construct new meta-stable brane configurations. For the second example, we add D6-branes more.

3.1 Electric theory

The type IIA brane configuration corresponding to $\mathcal{N}=1$ supersymmetric gauge theory with gauge group

$$SU(N_c) \times SU(N_c') \tag{3.1}$$

and the symmetric flavor for $SU(N_c)$, the conjugate symmetric flavor for $SU(N_c)$, a bifundamental X in the representation $(\mathbf{N_c}, \overline{\mathbf{N_c'}})$ and its conjugate field $\widetilde{X}$ in the representation $(\overline{\mathbf{N_c}}, \mathbf{N_c'})$, under the gauge group can be described similarly. It consists of a middle $NS5_M$-brane(012345), the left $NS5_L$-brane(012345) and the right $NS5_R$-brane(012345), the left $NS5_L'$-brane(012389) and the right $NS5_R'$-brane(012389), N_c- and N_c'-color D4-branes(01236) and an $O6^+$-plane(0123789). We take the arbitrary number of color D4-branes with the constraint $2N_c' \geq N_c$. The $O6^+$-plane acts as $(x^4, x^5, x^6) \to (-x^4, -x^5, -x^6)$ and has RR charge $+4$ playing the role of $+4$ D6-brane. The bifundamentals X and $\widetilde{X}$ correspond to 4-4 strings connecting the N_c-color D4-branes with N_c'-color D4-branes. The symmetric and conjugate symmetric flavors correspond to 4-4 strings connecting N_c D4-branes located at negative x^6 region and N_c D4-branes located at positive x^6 region. See also the relevant works in [15, 16, 17, 18].

The middle NS5-brane is located at $x^6 = 0$ and the x^6 coordinates for the $NS5_L$-brane, $NS5_L'$-brane, $NS5_R'$-brane and $NS5_R$-brane are given by $x^6 = -y_2, -y_1, y_1$ and $x^6 = y_2$ respectively, along the line of [2]. The N_c D4-branes are suspended between the $NS5_L'$-brane, whose x^6 coordinate is given by $x^6 = -y_1$, and $NS5_R'$-brane, whose x^6 coordinate is given by $x^6 = y_1$, while the N_c' D4-branes are suspended between the $NS5_L$-brane and the $NS5_L'$-brane and moreover they are suspended between the $NS5_R'$-brane and the $NS5_R$-brane. We draw this brane configuration in Figure 3A for the vanishing mass for the bifundamentals. See also the relevant previous work appeared in [19] [2].

The gauge couplings of $SU(N_c)$ and $SU(N_c')$ are given by

$$g_1^2 = \frac{g_s \ell_s}{y_1}, \qquad g_2^2 = \frac{g_s \ell_s}{y_2}. \tag{3.2}$$

[2]This is equivalent to the reduced brane configuration of [19] with particular rotations for the NS5-branes if we remove all the D6-branes completely.

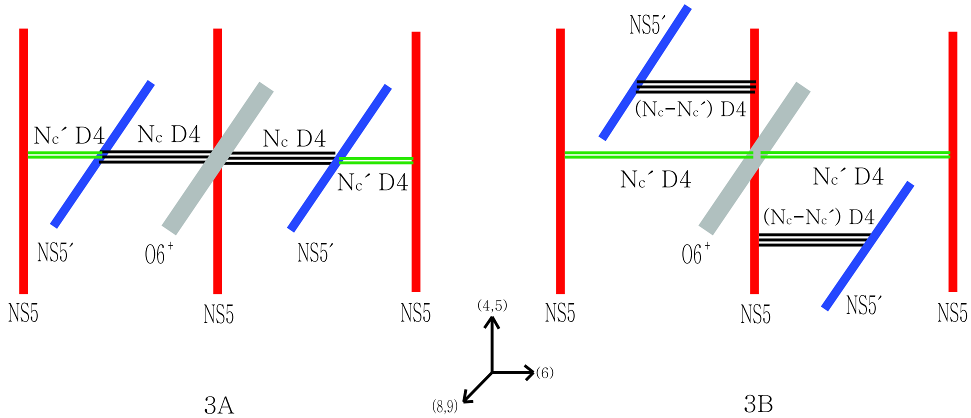

Figure 3: The $\mathcal{N}=1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N_c')$ and the bifundamentals X and $\widetilde{X}$ as well as symmetric and conjugate symmetric flavors with vanishing(3A) and nonvanishing(3B) mass for the bifundamentals corresponding to the dual for the second gauge group. The bifundamentals X and $\widetilde{X}$ correspond to 4-4 strings connecting the N_c-color D4-branes with N_c'-color D4-branes.

As y_2 goes to ∞, the $SU(N_c')$ gauge group becomes a global symmetry and the theory leads to SQCD-like theory with the gauge group $SU(N_c)$ with symmetric and conjugate symmetric flavors and N_c' flavors in the fundamental representation.

According to result of [19], there is no electric superpotential corresponding to the Figure 3A. Now let us deform this theory. Displacing the two NS5'-branes relative each other in the v direction corresponds to turning on a quadratic superpotential for the bifundamentals X and $\widetilde{X}$ as follows:

$$W = mX\widetilde{X} \, (\equiv m\Phi') \tag{3.3}$$

where the Φ' is a meson field and the mass m is given by geometrically

$$m = \frac{\Delta x}{2\pi\alpha'} \left(= \frac{\Delta x}{\ell_s^2} \right). \tag{3.4}$$

The $NS5'_L$-brane is moving to the $+v$ direction and the $NS5'_R$-brane is moving to $-v$ direction due to the O6-plane for fixed NS5-branes. That is, the x^5 coordinate of $NS5'_L$-brane is $+\Delta x$ while the x^5 coordinate of $NS5'_R$-brane is $-\Delta x$. We draw this brane configuration in Figure 3B for nonvanishing mass for the bifundamentals by moving the $NS5'_L$-brane with $(N_c - N_c')$ color D4-branes to the $+v$ direction and their mirrors to $-v$ direction. For the meta-stable brane configuration next subsection, we need to move outer NS5-branes rather than NS5'-branes.

3.2 Magnetic theory

Let us consider two separate cases.

• When the dual magnetic case is taken from the second gauge group

By applying the Seiberg dual to the $SU(N_c')$ factor in (3.1), the two $NS5'_{L,R}$-branes can be located at the outside of the three NS5-branes. Starting from Figure 3B and moving the $NS5'_R$-brane to the right all the way past the $NS5_R$-brane and then taking $\frac{\pi}{2}$ rotations of two outer NS5-branes, there exist the $\widetilde{N}_c'(= N_c - N_c')$ D4-branes that are connecting between two NS5'-brane(and their mirrors) and N_c D4-branes connecting between NS5'-brane and NS5-brane. Since $\widetilde{N}_c'$ is less than N_c, it is not possible to construct a misalignment of the flavor D4-branes. Therefore, there is no meta-stable brane configuration in this case.

• When the dual magnetic case is taken from the first gauge group

Starting from the Figure 3A, we apply the Seiberg dual to the $SU(N_c)$ factor in (3.1), the two NS5'-branes are interchanged each other. Then the number of color $\widetilde{N}_c$ is given by $\widetilde{N}_c = 2N_c' - N_c$ from [19, 20]. By rotating the outer two NS5-branes by $\frac{\pi}{2}$ and moving them to $\pm v$ direction, the N_c' D4-branes are connecting between two NS5'-branes(and their mirrors) and $\widetilde{N}_c$ D4-branes connecting between NS5'-brane and NS5-brane. By introducing N_c' D4-branes and N_c' anti-D4-branes between NS5'-brane and NS5-brane, reconnecting the former with the N_c' D4-branes connecting between two NS5'-branes and moving those combined D4-branes to v-direction(and their mirrors to $-v$ direction), one gets the final Figure 4A where we are left with $(N_c' - \widetilde{N}_c)$ anti-D4-branes between NS5'-brane and NS5-brane.

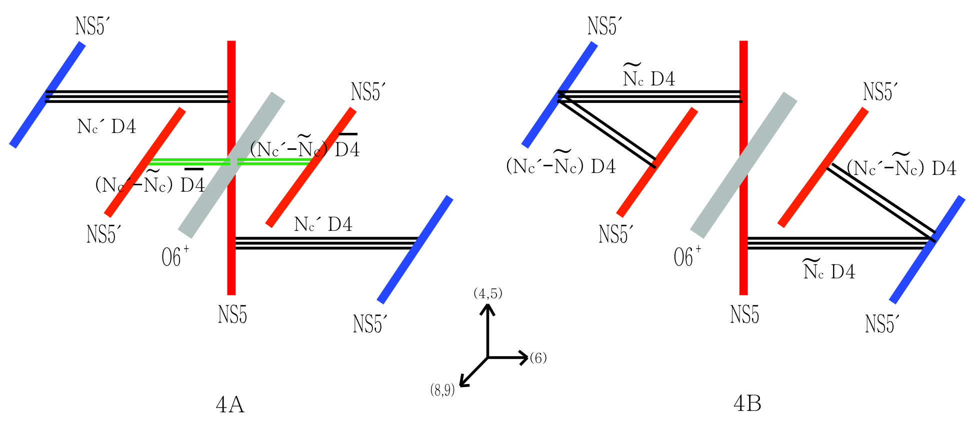

Figure 4: The magnetic brane configuration corresponding to Figure 3A with D4- and $\overline{D4}$-branes(4A) and with a misalignment between D4-branes(4B) when the NS5'-branes are close to each other.

The gauge group is given by

$$SU(\widetilde{N}_c = 2N_c' - N_c) \times SU(N_c') \tag{3.5}$$

where the number of dual color can be obtained from the linking number counting, as done in [19, 20]. The matter contents are the flavor singlet Y in the bifundamental representation $(\widetilde{\mathbf{N_c}}, \overline{\mathbf{N_c'}})$ and its complex conjugate field $\widetilde{Y}$ in the bifundamental representation $(\overline{\widetilde{\mathbf{N_c}}}, \mathbf{N_c'})$, under the dual gauge group (3.5) and the gauge singlet Φ' in the representation for $(\mathbf{1}, \mathbf{N_c'^2} - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the dual gauge group. There are also the symmetric flavor for $SU(\widetilde{N}_c)$ and the conjugate symmetric flavor for $SU(\widetilde{N}_c)$. A cubic superpotential is an interaction between dual "quarks" and a meson.

Then the dual magnetic superpotential, by adding the mass term like as (3.3) for the bifundamental X which can be interpreted as a linear term in the meson Φ' to this cubic superpotential, is given by

$$W_{dual} = \Phi' Y \widetilde{Y} + m\Phi' \tag{3.6}$$

where this can be seen from the equation (2.3) of [19] by putting the terms coming from the D6-branes in both electric and magnetic theories to zero. The brane configuration for zero mass for the bifundamentals can be obtained from Figure 4A by pushing the two NS5'-branes into the origin $v = 0$. Then the number of dual colors for D4-branes becomes N_c' between two NS5'-branes and $\widetilde{N}_c$ between the NS5'-brane and the NS5-brane.

The brane configuration in Figure 4A is stable as long as the distance Δx between the upper $NS5'$-brane and the middle $NS5'$-brane is large, as in [2]. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 4B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (3.5) and superpotential (3.6). When the two NS5'-branes which are connected by $\widetilde{N}_c$ D4-branes are replaced by two coincident D6-branes, the brane configuration of Figure 4B is the same as the one studied in [20, 4].

One can perform similar analysis in our brane configuration since one can take into account the behavior of parameters geometrically in the presence of O6-plane. Then the upper $(N_c' - \widetilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 4B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action, by following the procedure of [2]. Of course, their mirrors, the lower $(N_c' - \widetilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 4B can bend and their trajectory connecting two NS5'-branes should be preserved under the O6-plane, i.e., $\mathbf{Z}_2$ symmetric way, like as the symmetry property between the straight flavor D4-branes, when there is no gravitational interaction, under the O6-plane in Figure 4B. The correct choice for the ground state of the system depends on the parameters y_i and Δx.

In the remaining paragraphs, we describe DBI action very briefly since the main discussions

are done in previous subsection 2.2. Since the brane geometry of the upper tilted ($N_c' - \widetilde{N}_c = N_c - N_c'$) D4-branes, two NS5'-branes and NS5-brane in Figure 4B is exactly the same as the one in Figure 2B except that the corresponding number of D4-branes is different, the DBI analysis can be done straightforwardly by following the previous procedure. One can write down the DBI action as [2]

$$S(upper) = -(N_c - N_c')\tau_4 \int dx \sqrt{\frac{1}{H(y)} + (\partial_x y)^2}, \qquad H(y) \equiv 1 + \frac{\ell_s^2}{y^2} \tag{3.7}$$

where we inserted the correct number of D4-branes. The presence of N_c'-dependence(as well as N_c-dependence) in front of (3.7) comes from the fact that the number of dual colors depends on how one takes the Seiberg dual strictly [20]. This is a different aspect, compared with the one in [2] or the previous case considered in subsection 2.2. In this case also, there are a conserved quantity (2.6) and a relation by (2.7) because the number of D4-branes does not depend on these equations. The relative distance between the two NS5'-branes is characterized by (2.10). We rewrite here for convenience

$$\Delta x(upper) = x_2 = \frac{1}{2\ell_s}\left(y_1^2 \sin 2\theta_1 + y_2^2 \sin 2\theta_2\right) + \ell_s\left(\theta_1 + \theta_2\right). \tag{3.8}$$

Finally, the energy [2] of the Figure 4B with bending effect, coming from the upper tilted D4-branes, is written as

$$E_{curved}(upper) = (N_c - N_c')\tau_4 \frac{\sqrt{H(y_m)}}{2\ell_s}\left(y_1^2 \sin 2\theta_1 + y_2^2 \sin 2\theta_2\right) \tag{3.9}$$

where we also put the correct number of D4-branes for the present case and all the variables (y_i, y_m, θ_i) are the same as the one in subsection 2.2.

So far, we have only considered the contributions from the bending effect of upper tilted D4-branes in Figure 4B. Now we can compute the contributions from their mirrors, i.e., the lower tilted ($N_c - N_c'$) D4-branes in Figure 4B. The DBI action for these D4-branes, $S(lower)$, is the same as above $S(upper)$ since the number of D4-branes are the same and the background geometry is characterized by the same NS5-brane implying that the induced metric and dilaton are the same as before. That is,

$$S(lower) = S(upper) \tag{3.10}$$

with (3.7). It is straightfoward to see that there exist also a conserved quantity (2.6) and a relation (2.7). Note that under the replacement y_m by $-y_m$ which is the maximum value of y(note that the positive direction for y is directed to the left hand side of the NS5-brane

in Figure 4B), this relation (2.7) still holds. Now the coordinate x of lower NS5'-brane in Figure 4B is given by $x = -x_2$ while the coordinate y of it is given by $y = -y_2$. Recall that the O6-plane action reflects here also, as in Figure 4B. One can view the brane configuration consisting of the lower NS5'-branes and D4-branes in Figure 4B as the brane configuration after taking a reflection for lower NS5'-branes and D4-branes in Figure 2B with respect to the NS5-brane($y = 0$ plane). This procedure is equivalent to transform the y_i coordinates for NS5'-branes in Figure 2B as $-y_i$ keeping x coordinates unchanged. Or equivalently, the two bending curves for the upper and lower tilted D4-branes in Figure 4B are symmetric each other under the origin $(x, y) = (0, 0)$.

By using the separation of variables in (2.6), one can write the integrals for the two intervals $-x_m \leq x \leq 0$ and $-x_2 \leq x \leq -x_m$, as before. It turns out the results are given by (2.8) and (2.9) which is invariant under the $y_i \rightarrow -y_i$ and $y_m \rightarrow -y_m$. Moreover, the relative distance between two NS5'-branes, $\Delta x(lower) = 0 - (-x_2) = x_2$, is given by (3.8):

$$\Delta x(lower) = \Delta x(upper).$$

Since the energy of the Figure 4B with bending effect for the lower tilted D4-branes is given by minus $S(lower)$ together with (3.10), one obtains that

$$E_{curved}(lower) = E_{curved}(upper)$$

with (3.9). Note that although the functions $\Delta x(upper)$ and $E_{curved}(upper)$ depend on y_i and y_m, the replacements $y_i \rightarrow -y_i$ and $y_m \rightarrow -y_m$ does not change these functions. The $\mathbf{Z}_2$ symmetry by an O6-plane acting as $(x, y) \rightarrow (-x, -y)$ reflects here.

Then the analysis of [2] can be done using the results of (3.8) which is exactly the same as the one in [2] and (3.9) which has different overall coefficient containing the rank of the second gauge group N_c' due to the O6-plane when we compare with the unitary case, for equal y_i's and unequal y_i's. Once we understand the correct phase structure for the upper tilted D4-branes case which will be the same as the one [2] basically, then the corresponding analysis for its mirror, lower tilted D4-branes case is satisfied automatically.

3.3 Gauge theory analysis at small Δx

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (3.5) and the gauge couplings for the two gauge group factors are given by

$$g_{1,mag}^2 = \frac{g_s \ell_s}{y_1}, \qquad g_{2,mag}^2 = \frac{g_s \ell_s}{y_2 - y_1}.$$

The dual gauge theory has an adjoint Φ' of $SU(N_c')$ and bifundamental Y in the representation $(\widetilde{\mathbf{N_c}}, \overline{\mathbf{N_c'}})$ under the dual gauge group (3.5) and the superpotential corresponding to Figures 4A and 4B is given by

$$W_{dual} = h\Phi' Y\widetilde{Y} - h\mu^2\Phi', \qquad h^2 = g_{2,mag}^2$$

and the mass parameter μ^2 is given by (2.15). Then $Y\widetilde{Y}$ is a $\widetilde{N}_c \times \widetilde{N}_c$ matrix where the second gauge group indices for Y and $\widetilde{Y}$ are contracted with those of Φ' while μ^2 is a $N_c' \times N_c'$ matrix. Although the field Y itself is a fundamental in the second gauge group which is a different feature, compared with the singlet representation for the usual quark coming from D6-branes [19], the product $Y\widetilde{Y}$ has the same representation with the product, $q\widetilde{s}s\widetilde{q}$ in the notation of [19]. Moreover, the second gauge group indices for the field Φ' play the role of the flavor indices for the gauge singlet $M' \equiv Q\widetilde{Q}$ in [19].

Therefore, the F-term equation, the derivative W_{dual} with respect to the meson field Φ' cannot be satisfied if the N_c' exceeds $\widetilde{N}_c$. So the supersymmetry is broken. That is, there are three equations from F-term conditions: $Y\widetilde{Y} - \mu^2 = 0, \Phi' Y = 0$, and $\widetilde{Y}\Phi' = 0$. Then the solutions for these are given by

$$< Y >= \left(\begin{array}{c} \mu e^{\phi} \mathbf{1}_{\widetilde{N}_c} \\ 0 \end{array} \right), \quad < \widetilde{Y} >= \left(\begin{array}{cc} \mu e^{-\phi} \mathbf{1}_{\widetilde{N}_c} & 0 \end{array} \right), \quad < \Phi' >= \left(\begin{array}{cc} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{(N_c' - \widetilde{N}_c)} \end{array} \right).$$

At one loop, the effective potential $V_{eff}^{(1)}$ for Φ_0 leads to the positive value for $m_{\Phi_0}^2$ implying that these vacua are stable. The gauge theory analysis where the theory will be strongly coupled in the IR region $N_c' > 2\widetilde{N}_c - 2$ is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C.

3.4 Other electric and magnetic theories with same gauge group and different matters

The type IIA brane configuration corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (3.1) and the antisymmetric flavor for $SU(N_c)$, the conjugate symmetric flavor for $SU(N_c)$, eight fundamentals for $SU(N_c)$, a bifundamental X in the representation $(\mathbf{N_c}, \overline{\mathbf{N_c'}})$ and its conjugate field $\widetilde{X}$ in the representation $(\overline{\mathbf{N_c}}, \mathbf{N_c'})$, under the gauge group can be described similarly. It consists of a middle $NS5'_M$-brane, the left $NS5'_L$-brane and the right $NS5'_R$-brane, the left $NS5_L$-brane and the right $NS5_R$-brane, N_c- and N_c'-color D4-branes, eight semi-infinite D6-branes, an $O6^+$-plane and $O6^-$-plane.

The middle NS5'-brane is located at $x^6 = 0$ and the x^6 coordinates for the $NS5'_L$-brane, $NS5_L$-brane, $NS5_R$-brane and $NS5'_R$-brane are given by $x^6 = -y_2, -y_1, y_1$ and $x^6 = y_2$

respectively. The N_c D4-branes are suspended between the $NS5_L$-brane, whose x^6 coordinate is given by $x^6 = -y_1$, and $NS5_R$-brane, whose x^6 coordinate is given by $x^6 = y_1$, while the N_c' D4-branes are suspended between the $NS5_L$-brane and the $NS5'_L$-brane and further they are suspended between the $NS5'_R$-brane and the $NS5_R$-brane. We draw this brane configuration in Figure 5A for the vanishing mass for the bifundamentals. See also the relevant previous work appeared in [19] [3]. The gauge couplings of $SU(N_c)$ and $SU(N_c')$ are given by (3.2), as before. See also the relevant works in [21, 22, 23].

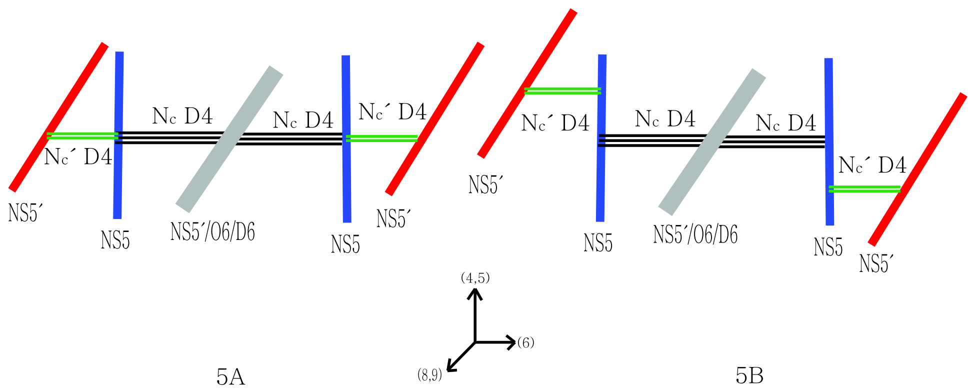

Figure 5: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N_c')$ and the bifundamentals X and $\widetilde{X}$ as well as antisymmetric, conjugate symmetric flavors and eight D6-branes with vanishing(5A) and nonvanishing(5B) mass for the bifundamentals.

There is no electric superpotential corresponding to the Figure 5A. Now let us deform this theory. Displacing the two NS5'-branes relative each other in the v direction corresponds to turning on a quadratic superpotential for the bifundamentals X and $\widetilde{X}$ as (3.3) where the Φ' is a meson field and the mass m is given by (3.4). The $NS5'_L$-brane is moving to the $+v$ direction and the $NS5'_R$-brane is moving to $-v$ direction due to the O6-plane for fixed NS5-branes. In other words, the x^5 coordinate of $NS5'_L$-brane is $+\Delta x$ while the x^5 coordinate of $NS5'_R$-brane is $-\Delta x$. We draw this brane configuration in Figure 5B for nonvanishing mass for the bifundamentals by moving the $NS5'_L$-brane with N_c' color D4-branes to the $+v$ direction and their mirrors to $-v$ direction.

Let us apply the Seiberg dual to the $SU(N_c)$ factor. Starting from Figure 5B and moving the $NS5_L$-brane to the right all the way past the $NS5'_M$-brane(and $NS5_R$-brane to the left of $NS5'_M$-brane), one obtains the Figure 6A. By introducing N_c' D4-branes and N_c' anti-D4-branes between $NS5_R$-brane and $NS5'_M$-brane, we are left with $(N_c' - \widetilde{N}_c)$ anti-D4-branes

[3] This is equivalent to the reduced brane configuration in section 4 of [19] with particular rotations for the NS5-branes if we remove all the D6-branes completely.

between $NS5_R$-brane and $NS5'_M$-brane. The brane configuration for zero mass for the bifundamental can be obtained from Figure 6A by pushing N'_c D4-branes into the origin $v = 0$.

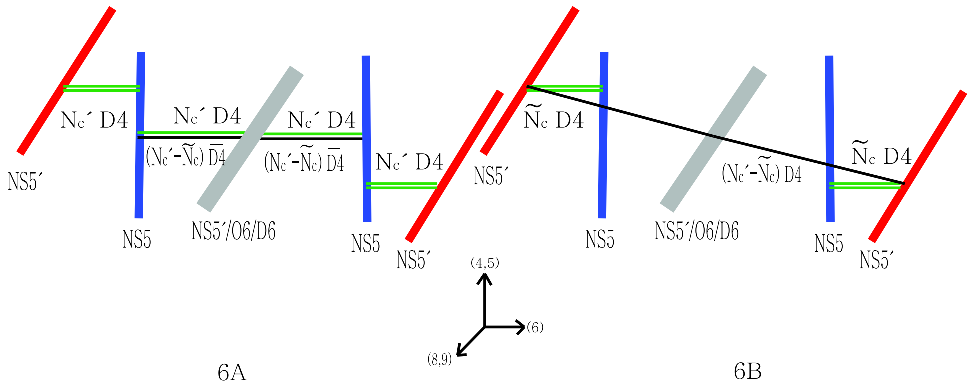

Figure 6: The magnetic brane configuration corresponding to Figure 5B with D4- and $\overline{D4}$-branes(6A) and with a misalignment between D4-branes(6B) when the NS5'-branes are close to each other.

The gauge group is given by

$$SU(\widetilde{N}_c = 2N'_c - N_c + 4) \times SU(N'_c) \tag{3.11}$$

where the number of dual color can be obtained from the linking number counting, as done in [19, 24]. The matter contents are the flavor singlet Y in the bifundamental representation $(\widetilde{\mathbf{N}}_{\mathbf{c}}, \overline{\mathbf{N}'_{\mathbf{c}}})$ and its complex conjugate field $\widetilde{Y}$ in the bifundamental representation $(\overline{\widetilde{\mathbf{N}}_{\mathbf{c}}}, \mathbf{N}'_{\mathbf{c}})$, and the gauge singlet $\Phi' \equiv X\widetilde{X}$ in the representation for $(\mathbf{1}, \mathbf{N'^2_c} - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$, under the dual gauge group. There are also the antisymmetric flavor a, the conjugate symmetric flavor $\widetilde{s}$ and eight fundamentals $\hat{q}$ for $SU(\widetilde{N}_c)$.

Then the dual magnetic superpotential, by adding the mass term for the bifundamental X, is given by

$$W_{dual} = \Phi' Y \widetilde{Y} + m\Phi' + \hat{q}\widetilde{s}\hat{q} \tag{3.12}$$

where this can be seen from the equation (4.2) of [19] by putting the terms coming from the D6-branes in both electric and magnetic theories to zero.

The brane configuration in Figure 6A is stable as long as the distance Δx between the upper $NS5'_L$-brane and the middle $NS5'_M$-brane is large. If they are close to each other then this brane configuration is unstable to decay and it becomes the brane configuration in Figure 6B. Since the two NS5'-branes are located at different sides of NS5-brane in Figure 6B, contrary to the previous cases, the x^6 coordinates for NS5'-branes are positive and negative when we take $x^6 = 0$ for the NS5-brane. For the DBI computation, this fact should be taken

into account. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (3.11) and superpotential (3.12). When the two NS5'-branes which are connected by $\widetilde{N}_c$ D4-branes are replaced by two coincident D6-branes, the brane configuration of Figure 6B is the same as the one studied in [24, 4].

The gauge couplings for the two gauge group factors are given by

$$g_{1,mag}^2 = \frac{g_s \ell_s}{y_1}, \qquad g_{2,mag}^2 = \frac{g_s \ell_s}{(y_2 - y_1)}$$

and the superpotential corresponding to Figures 6A and 6B is given by

$$W_{dual} = h\Phi' Y\widetilde{Y} - h\mu^2 \Phi' + \hat{q}\widetilde{s}\hat{q}, \qquad h^2 = g_{2,mag}^2$$

and the mass parameter μ^2 is given by (2.15). Then $Y\widetilde{Y}$ is a $\widetilde{N}_c \times \widetilde{N}_c$ matrix where the second gauge group indices for Y and $\widetilde{Y}$ are contracted with those of Φ' while μ^2 is a $N_c' \times N_c'$ matrix. Although the field Y itself is a fundamental in the second gauge group which is a different feature, compared with the singlet representation for the usual quark coming from D6-branes [19], the product $Y\widetilde{Y}$ has the same representation with the product of dual quarks, $q\widetilde{s}a\widetilde{q}$ in the notation of [19]. Moreover, the second gauge group indices for the field Φ' play the role of the flavor indices for the gauge singlet $M' \equiv Q\widetilde{Q}$ in [19].

Therefore, the F-term equation, the derivative W_{dual} with respect to the meson field Φ' cannot be satisfied if the N_c' exceeds $\widetilde{N}_c$. So the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. That is, there are five equations from F-term conditions: $Y\widetilde{Y} - \mu^2 = 0, \Phi' Y = 0, \widetilde{Y}\Phi' = 0, \hat{q}\widetilde{s} = 0$, and $\hat{q}\hat{q} = 0$. Then the solutions for these are given by

$$\begin{aligned} < Y > &= \begin{pmatrix} \mu e^{\phi} \mathbf{1}_{\widetilde{N}_c} \\ 0 \end{pmatrix}, \quad < \widetilde{Y} >= \begin{pmatrix} \mu e^{-\phi} \mathbf{1}_{\widetilde{N}_c} & 0 \end{pmatrix}, \quad < \Phi' >= \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{(N_c' - \widetilde{N}_c)} \end{pmatrix}, \\ < \hat{q} > &= 0, \quad < \widetilde{s} >= 0. \end{aligned}$$

One can expand around the solutions. Although there exists an extra last term in (3.12), this does not contribute to the one loop result. At one loop, the effective potential $V_{eff}^{(1)}$ for Φ_0 leads to the positive value for $m_{\Phi_0}^2$ implying that these vacua are stable. The gauge theory analysis where the theory will be strongly coupled in the IR region $N_c' > 2\widetilde{N}_c - 4$ is only valid in the regime where Δx is smaller than $\exp(-\frac{C}{g_s})$ with some positive constant C as before.

4 Conclusions and outlook

The meta-stable brane configurations we have found are summarized by Figures 2, 4, and 6. If we replace the upper and lower NS5'-branes in Figures 2B, 4B and 6B with the coinci-

dent D6-branes, those brane configurations become nonsupersymmetric minimal energy brane configurations in [4, 13], in [4, 20], and in [4, 24] respectively.

It would be interesting to construct the meta-stable brane configuration where there exist four NS5-branes by adding one extra outer NS5-brane to the brane configuration found in [2] or to the brane configuration of Figure 1 in this paper or to construct the meta-stable brane configuration where there exist six NS5-branes by adding two extra outer NS5-branes to the brane configuration found in [25]. Or one can add two extra outer NS5-branes to the brane configuration found in [19]. These gauge theories will be triple product gauge group theories.

Some different directions concerning on the meta-stable vacua in different contexts are present in recent works [26]-[36] where some of them use anti D-branes and some of them are described in the type IIB theory. It would be very interesting to find out how the meta-stable brane configurations from type IIA string theory and those brane configuration from type IIB theory are related to each other.

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